Rational Equations and Functions

Algebra II

Chapter 8

Algebra II 8

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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8.1 Model Inverse and Joint Variation

- Direct Variation: y = ax
  - x ↑, y ↑
- Inverse Variation: $y = \frac{a}{x}$
  - x ↑, y ↓
- Joint Variation: y = axz
  - y depends on both x and z

a
is the
constant of
variation

8.1 Model Inverse and Joint Variation

- What type of variation is each of the following?
  - $xy = 48$
  - $2y = x$
  - $y = 2x + 3$

$y = 48 / x$ → inverse

$y = ½ x$ → direct

+3 means neither

8.1 Model Inverse and Joint Variation

- Solving Variations
  - Plug in x and y to find a
  - Plug in a and the other value and solve

- y varies inversely as x. When x = 2, y = 6. Write an equation relating x and y. Then find y when x = 4.

$y = k/x \rightarrow 6 = k/2 \rightarrow 12 = k$
$y = 12 / 4 \rightarrow y = 3$

8.1 Model Inverse and Joint Variation

- Checking data for variation
  - Plug each of the data points in one of the variation equations to find a
  - If the a stays the same, the data has that type of variation
- What type of variation?

X	2	4	8
y	8	4	2

8.1 Model Inverse and Joint Variation

- Writing variations from sentences
  - y varies directly with x and inversely with z^2
  - z varies jointly with x^2 and y
  - y varies inversely with x and z

Varies means "=a"

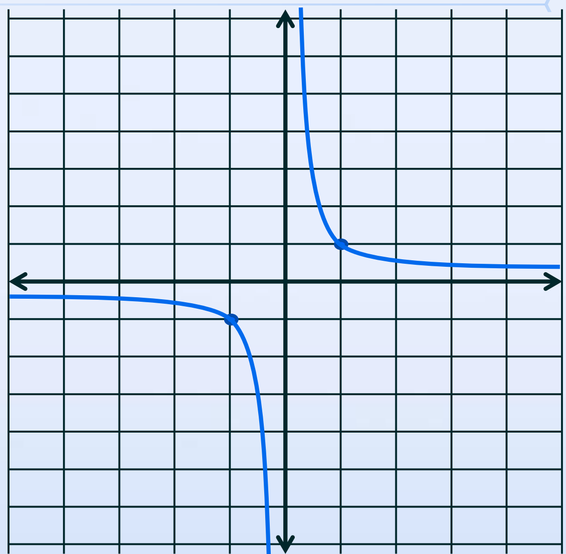
$y = ax/z^2$
$z = ax^2y$
$y = axz$

Quiz

- 8.1 Homework Quiz

8.2 Graph Simple Rational Functions

- Rational Functions
  - Functions written as a fraction with x in the denominator
  - $y = \frac{1}{x}$
- Shape called hyperbola
- Asymptotes
  - Horizontal: x-axis
  - Vertical: y-axis



8.2 Graph Simple Rational Functions

- General form
  - $y = \frac{a}{x-h} + k$
    - a → stretches vertically (multiplies y-values)
    - h → moves right
    - k → moves up
- How is $y = \frac{2}{x+3} + 4$ transformed from $y = \frac{1}{x}$?

Stretches vertically by factor of 2
Moves left 3
Moves up 4

8.2 Graph Simple Rational Functions

- How to find asymptotes
  - Vertical
    - Make the denominator = 0 and solve for x

Vertical: $3x - 6 = 0 \rightarrow 3x = 6 \rightarrow x = 2$
Horizontal: $y = (2*1000000)/(3*1000000 - 6) \rightarrow y = 2/3$

8.2 Graph Simple Rational Functions

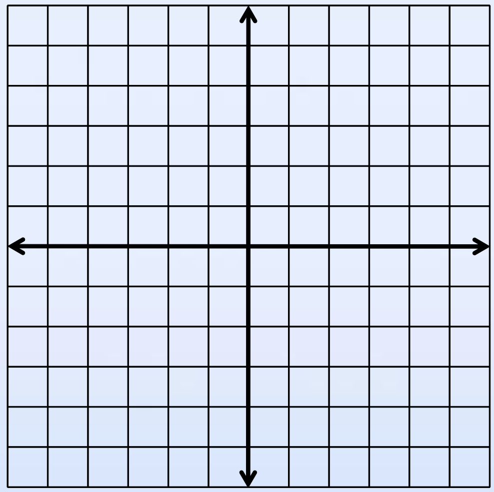
- Horizontal
  - Substitute a very large number for x and estimate y
- Or
  - Find the degree of numerator (N)
  - Find the degree of denominator (D)
  - If N < D, then y = 0
  - If N = D, then y = leading coefficients
  - If N > D, then no horizontal asymptote
- Find the asymptotes for $y = \frac{2x}{3x-6}$

8.2 Graph Simple Rational Functions

- Domain
  - All x's except for the vertical asymptotes
- Range
  - All the y's covered in the graph
  - Usually all y's except for horizontal asymptote

8.2 Graph Simple Rational Functions

- Graph by finding asymptotes and making a table
- Graph $y = \frac{2}{x+3} + 4$



Asymptotes

Vertical: x + 3 = 0 → x = -3

Horizontal: y = 2/(1000000 + 3) + 4→ y = 4

Quiz

- 8.2 Homework Quiz

8.3 Graph General Rational Functions

- Find the asymptotes
  - Simplify first
    - Factor and cancel entire factors
  - Vertical
    - take the denominator = 0 and solve for x

Vertical: $x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1$
Horizontal: $y = (2(1000000)^2 + 1000000)/(1000000^2 - 1) \rightarrow y = 2$

8.3 Graph General Rational Functions

- Horizontal
  - Substitute a very large number for x and estimate y
- Or
  - Find the degree of numerator (N)
  - Find the degree of denominator (D)
  - If N < D, then y = 0
  - If N = D, then y = leading coefficients
  - If N > D, then no horizontal asymptote

- Find the asymptotes for $y = \frac{2x^2+x}{x^2-1}$

8.3 Graph General Rational Functions

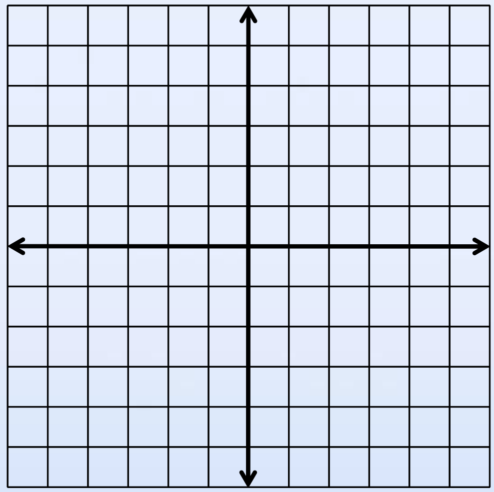
- How to find x-intercepts
  - Let $y = 0$
    - If $y = \frac{numerator}{denominator} = 0$
    - Only happens if numerator = 0
- How to find y-intercepts
  - Let $x = 0$ and simplify

8.3 Graph General Rational Functions

- To graph rational functions
  - Find the asymptotes
  - Make a table of values around the vertical asymptotes
  - Graph the asymptotes and points
  - Start near an asymptote, go through the points and end near another asymptote
    - Each graph will have several sections
    - NEVER cross a vertical asymptote

8.3 Graph General Rational Functions

- Graph $y = \frac{2x^2+x}{x^2-1}$



Vertical: $x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1$
Horizontal: $y = (2(1000000)^2 + 1000000)/(1000000^2 - 1) \rightarrow y = 2$

Quiz

- 8.3 Homework Quiz

8.4 Multiply and Divide Rational Expressions

- Simplified form → numerator and denominator can have no common factors

- Steps to simplify
  - Factor numerator and denominator
  - Cancel any common factors

8.4 Multiply and Divide Rational Expressions

Simplify

- $\frac{x^2-5x-6}{x^2-1}$
- $\frac{x^3+5x^2+6x}{x^3+2x^2}$

((x-6)(x+1))/((x-1)(x+1)) → (x-6)/(x-1)

(x(x+3)(x+2))/(x^2(x+2)) → (x+3)/x

8.4 Multiply and Divide Rational Expressions

- Multiplying Rational Expressions
  - Factor numerators and denominators
  - Multiply across top and bottom
  - Cancel factors

8.4 Multiply and Divide Rational Expressions

- $\frac{3x-27x^3}{3x^2-2x-1} \cdot \frac{3x^2-4x+1}{3x}$
- $\frac{x+2}{27x^3+8} \cdot (9x^2-6x+4)$

(-3x(9x²-1))/((3x+1)(x-1)) * ((3x-1)(x-1)) → (-3x(3x-1)(3x+1)(3x-1)(x-1))/(3x(3x+1)(x-1)) → -(3x-1)²

(x+2)/((3x+2)(9x²-6x+4)) * (9x²-6x+4)/1 → (x+2)/(3x+2)

8.4 Multiply and Divide Rational Expressions

- Dividing Rational Expressions
  - Take reciprocal of divisor
  - Multiply
- $\frac{3}{4x-8} \div \frac{x^2+3x}{x^2+x-6}$

3/(4x-8) * (x^2+x-6)/(x^2+3x) → 3/(4(x-2)) * ((x-2)(x+3))/(x(x+3)) → 3/(4x)

8.4 Multiply and Divide Rational Expressions

- Combined Operations
  - Do the first two operations
  - Use that result with the next operation

Quiz

- 8.4 Homework Quiz

8.5 Add and Subtract Rational Expressions

- Adding and Subtracting
  - Need least common denominator (LCD)
    - If LCD already present, add or subtract numerators only
  - To get fractions with LCD
    - Factor all denominators
    - LCD is the common factors times the unique factors
    - Whatever you multiply the denominator by, multiply the numerator also

8.5 Add and Subtract Rational Expressions

- $\frac{3}{2x} - \frac{7}{2x}$
- $\frac{3x}{x-4} + \frac{6}{x-4}$

-4/(2x) → -2/x

(3x+6)/(x-4)

8.5 Add and Subtract Rational Expressions

- $\frac{4}{3x^2} + \frac{x}{6x^3+3x^2}$
- $\frac{x+1}{x^2+6x+9} - \frac{1}{x^2-9}$

$$\frac{4}{3x^2} + \frac{x}{3x^2(2x+1)} \rightarrow \frac{4(2x+1)}{3x^2(2x+1)} + \frac{x}{3x^2(2x+1)} \rightarrow \frac{9x+4}{3x^2(2x+1)}$$

$$\frac{x+1}{(x+3)(x+3)} - \frac{1}{(x+3)(x-3)} \rightarrow \frac{(x+1)(x-3)}{(x+3)^2(x-3)} - \frac{x+3}{(x+3)^2(x-3)}$$

$$\rightarrow \frac{x^2-2x-3}{(x+3)^2(x-3)} - \frac{x+3}{(x+3)^2(x-3)} \rightarrow \frac{x^2-3x-6}{(x+3)^2(x-3)}$$

8.5 Add and Subtract Rational Expressions

- Simplifying Complex Fractions
  - Fractions within fractions
  - Follow order of operations (groups first)
  - Divide

8.5 Add and Subtract Rational Expressions

- $$\frac{\frac{3}{x-4}}{\frac{1}{x-4}+\frac{3}{x+1}}$$

$$\frac{\frac{3}{x-4}}{\frac{1}{x-4}+\frac{3}{x+1}} \rightarrow \frac{\frac{3}{x-4}}{\frac{1(x+1)}{(x-4)(x+1)}+\frac{3(x-4)}{(x-4)(x+1)}}$$

$$\rightarrow \frac{\frac{3}{x-4}}{\frac{x+1}{(x-4)(x+1)}+\frac{3x-12}{(x-4)(x+1)}} \rightarrow \frac{\frac{3}{x-4}}{\frac{4x-11}{(x-4)(x+1)}} \rightarrow \frac{3}{x-4} \cdot \frac{(x-4)(x+1)}{4x-11}$$

$$\rightarrow \frac{3(x+1)}{4x-11}$$

Quiz

- 8.5 Homework Quiz

8.6 Solve Rational Equations

- Only when the = sign is present!!!
- Method 1: simplify both sides and cross multiply
- Method 2:
- Multiply both sides by LCD to remove fractions
- Solve
- Check answers

8.6 Solve Rational Equations

- $\frac{3}{x} - \frac{1}{2} = \frac{12}{x}$
- $\frac{5x}{x+1} = 4 - \frac{5}{x+1}$

(3(2x))/x – 2x/2 = (12(2x))/x → 6 – x = 24 → -x = 18 → x = -18

5x = 4(x+1) – 5 → 5x = 4x + 4 – 5 → x = -1
Check answer: can’t divide by -1 so NO SOLUTION

8.6 Solve Rational Equations

- $\frac{3x-2}{x-2} = \frac{6}{x^2-4} + 1$
- $\frac{3}{x^2+4x} = \frac{1}{x+4}$

(3x-2)/(x-2) = 6/((x-2)(x+2)) + 1 → (3x-2)(x+2) = 6 + (x-2)(x+2) → $3x^2 + 4x - 4 = 6 + x^2 - 4$ → $2x^2 + 4x - 6 = 0$ → $x^2 + 2x - 3 = 0$ → $(x - 1)(x + 3) = 0$ → x = 1, -3

3/(x(x+4)) = 1/(x+4) → 3 = x

Quiz

- 8.6 Homework Quiz